Response approach to the integrated shear 3-point correlation function

arxiv:2102.10177 , **arxiv:2201.05607**

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In collaboration with: Stella Seitz, Oliver Friedrich, **Alex Barreira** and others!

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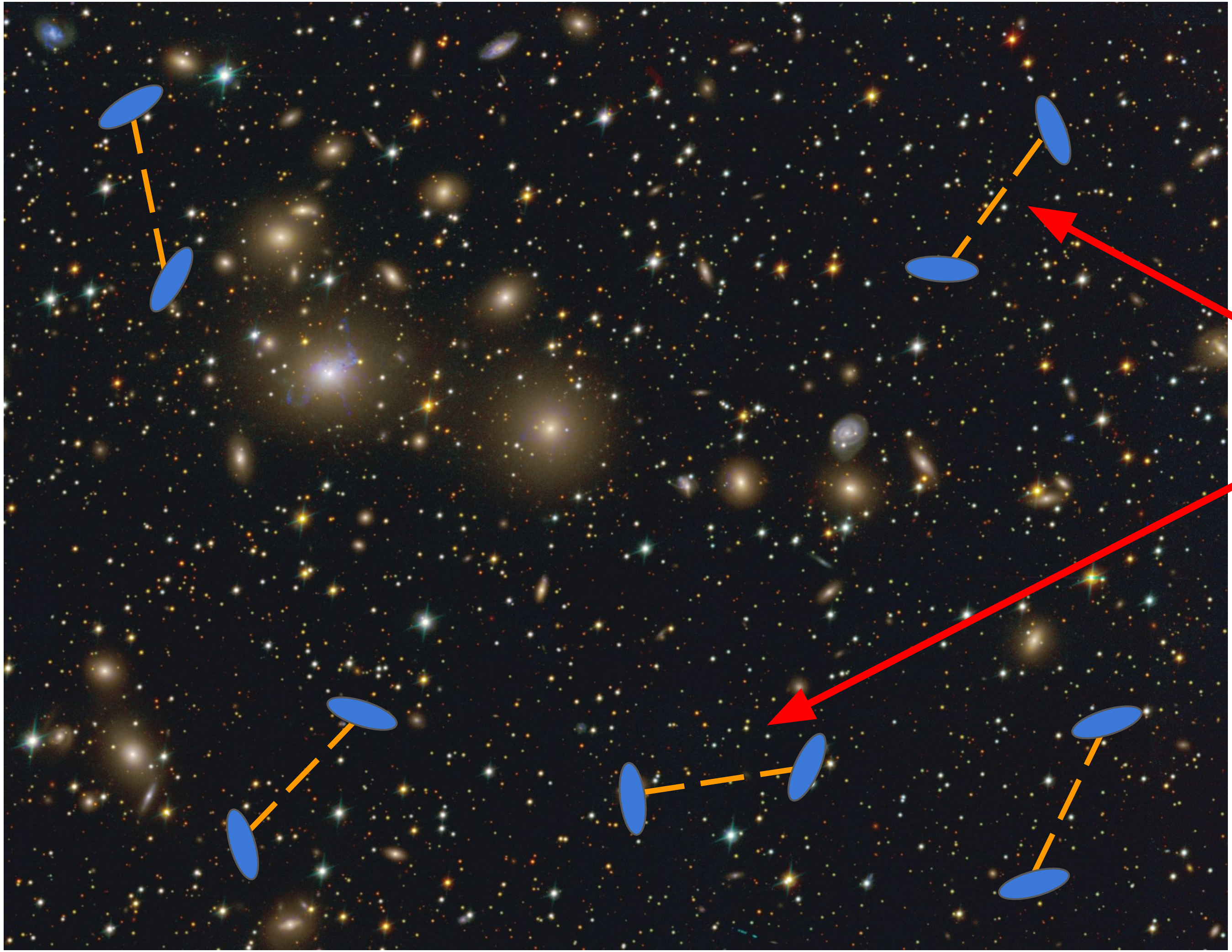
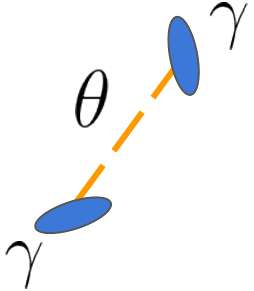
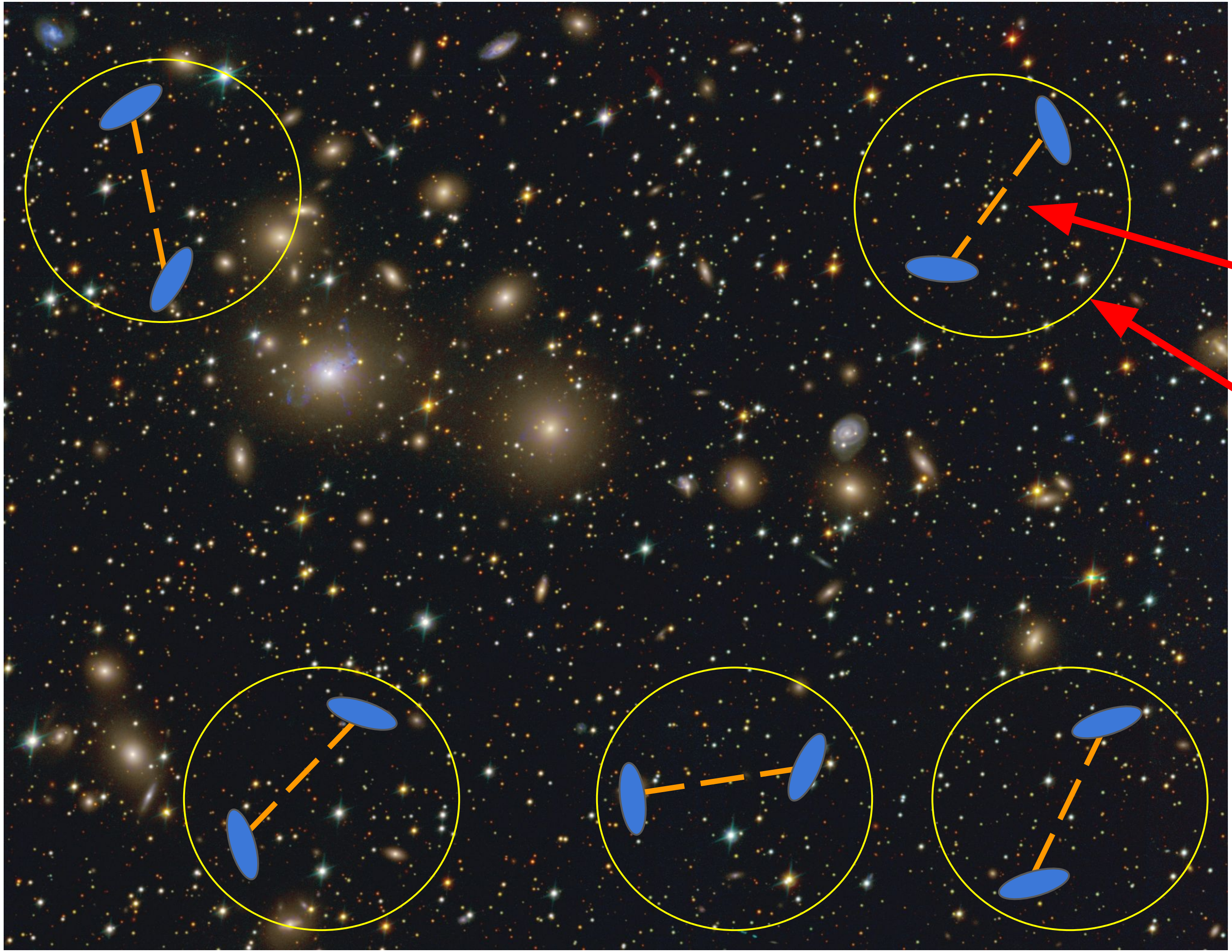




$$\hat{\xi}_{\pm}(\theta)$$

2-point shear correlation functions

Probes the projection of the 3D matter power spectrum



Position-dependent (local) shear 2-pt correlation in top-hat aperture **W**

$$\hat{\xi}_{\pm}(\theta;\boldsymbol{\theta_C})$$

Aperture mass

$$M_{\rm ap}(\boldsymbol{\theta_C})$$

Weighted tangential shear inside a compensated filter **U**

Integrated shear 3-point correlation function (3PCF)

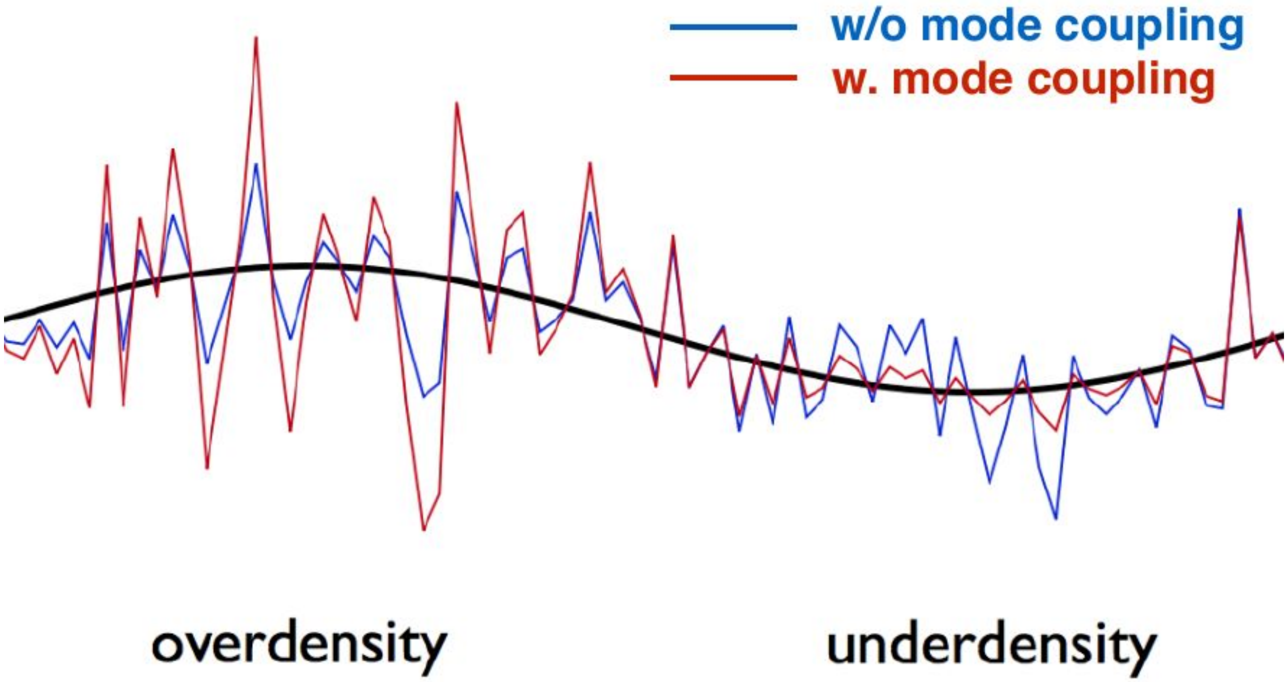
$$\zeta_{\pm}(\theta) \equiv \left\langle M_{\mathrm{ap}}(\boldsymbol{\theta}_C)\, \hat{\xi}_{\pm}(\theta; \boldsymbol{\theta}_C) \right\rangle$$

Direct observables of the shear field

- Higher order statistic of the cosmic shear field

- Probes an integrated quantity of the projected 3D matter bispectrum (or 3-point correlation function)

Intuitively …



Source: C. T. Chiang, E. Komatsu

Modelling the integrated shear 3PCF

Shear 2PCFs

$$\xi_{+,\mathrm{gh}}(\alpha) = \int \frac{\mathrm{d}l\ l}{2\pi}\ P_{\kappa,\mathrm{gh}}(l)\ J_0(l\alpha)$$

$$\xi_{-,\mathrm{gh}}(\alpha) = \int \frac{\mathrm{d}l\ l}{2\pi}\ P_{\kappa,\mathrm{gh}}(l)\ J_4(l\alpha)$$

Integrated shear 3PCFs

$$\zeta_{\pm,\mathrm{fgh}}(\boldsymbol{\alpha}) \equiv \left\langle M_{\mathrm{ap,f}}(\boldsymbol{\theta}_C)\ \hat{\xi}_{\pm,\mathrm{gh}}(\boldsymbol{\alpha};\boldsymbol{\theta}_C)\right\rangle$$

$$\zeta_{+,\mathrm{fgh}}(\alpha) = \frac{1}{A_{2\mathrm{pt}}(\alpha)} \int \frac{\mathrm{d}l\ l}{2\pi}\ \mathcal{B}_{+,\mathrm{fgh}}(l)\ J_0(l\alpha)$$

$$\zeta_{-,\mathrm{fgh}}(\alpha) = \frac{1}{A_{2\mathrm{pt}}(\alpha)} \int \frac{\mathrm{d}l\ l}{2\pi}\ \mathcal{B}_{-,\mathrm{fgh}}(l)\ J_4(l\alpha)$$

Shear 2PCFs

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$$P_{\kappa,\mathrm{gh}}(l) = \int \mathrm{d}\chi \frac{q_{\mathrm{g}}(\chi) q_{\mathrm{h}}(\chi)}{\chi^2} P_{\delta}^{\mathrm{3D}}\left(k = \frac{l}{\chi}, \chi\right)$$

Integrated shear 3PCFs

$$\zeta_{\pm,\mathrm{fgh}}(\boldsymbol{\alpha}) \equiv \left\langle M_{\mathrm{ap,f}}(\boldsymbol{\theta}_C)\ \hat{\xi}_{\pm,\mathrm{gh}}(\boldsymbol{\alpha};\boldsymbol{\theta}_C)\right\rangle$$

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$$\begin{aligned}\mathcal{B}_{\pm,\mathrm{fgh}}(\boldsymbol{l}) = &\int \mathrm{d}\chi \frac{q_{\mathrm{f}}(\chi) q_{\mathrm{g}}(\chi) q_{\mathrm{h}}(\chi)}{\chi^4} \int \frac{\mathrm{d}^2\boldsymbol{l}_1}{(2\pi)^2} \int \frac{\mathrm{d}^2\boldsymbol{l}_2}{(2\pi)^2} \\ &\times\ B_{\delta}^{\mathrm{3D}}\left(\frac{\boldsymbol{l}_1}{\chi}, \frac{\boldsymbol{l}_2}{\chi}, \frac{-\boldsymbol{l}_1 - \boldsymbol{l}_2}{\chi}, \chi\right) e^{2i(\phi_2 \mp \phi_{-1-2})} \\ &\times\ U(\boldsymbol{l}_1) W(\boldsymbol{l}_2 + \boldsymbol{l}) W(-\boldsymbol{l}_1 - \boldsymbol{l}_2 - \boldsymbol{l})\ .\end{aligned}$$

Shear 2PCFs

$$\xi_{+,\mathrm{gh}}(\alpha) = \int \frac{\mathrm{d}l\, l}{2\pi} P_{\kappa,\mathrm{gh}}(l)\, J_0(l\alpha)$$

$$\xi_{-,\mathrm{gh}}(\alpha) = \int \frac{\mathrm{d}l\, l}{2\pi} P_{\kappa,\mathrm{gh}}(l)\, J_4(l\alpha)$$

$$P_{\kappa,\mathrm{gh}}(l) = \int \mathrm{d}\chi \frac{q_\mathrm{g}(\chi) q_\mathrm{h}(\chi)}{\chi^2} \boxed{P_\delta^{3\mathrm{D}}\left(k = \frac{l}{\chi}, \chi\right)}$$

3D matter power spectrum

Integrated shear 3PCFs

$$\zeta_{\pm,\mathrm{fgh}}(\boldsymbol{\alpha}) \equiv \left\langle M_{\mathrm{ap,f}}(\boldsymbol{\theta}_C)\, \hat{\xi}_{\pm,\mathrm{gh}}(\boldsymbol{\alpha};\boldsymbol{\theta}_C)\right\rangle$$

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3D matter bispectrum

Large $\boldsymbol{l}$ limit of the integrated shear bispectrum

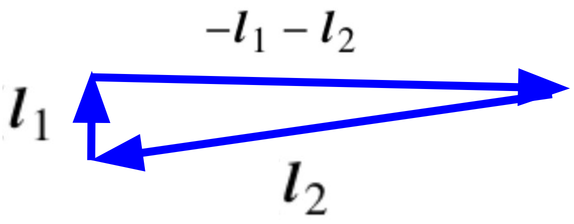
$$\mathcal{B}_{\pm,\mathrm{fgh}}(\boldsymbol{l}) = \int \mathrm{d}\chi \frac{q_\mathrm{f}(\chi)q_\mathrm{g}(\chi)q_\mathrm{h}(\chi)}{\chi^4} \int \frac{\mathrm{d}^2\boldsymbol{l}_1}{(2\pi)^2} \int \frac{\mathrm{d}^2\boldsymbol{l}_2}{(2\pi)^2}$$
$$\times\ B_\delta^{\mathrm{3D}}\left(\frac{\boldsymbol{l}_1}{\chi}, \frac{\boldsymbol{l}_2}{\chi}, \frac{-\boldsymbol{l}_1-\boldsymbol{l}_2}{\chi}, \chi\right) e^{2i(\phi_2 \mp \phi_{-1-2})}$$
$$\times\ U(\boldsymbol{l}_1)W(\boldsymbol{l}_2+\boldsymbol{l})W(-\boldsymbol{l}_1-\boldsymbol{l}_2-\boldsymbol{l})\ .$$

Large $\boldsymbol{l}$ limit of the integrated shear bispectrum

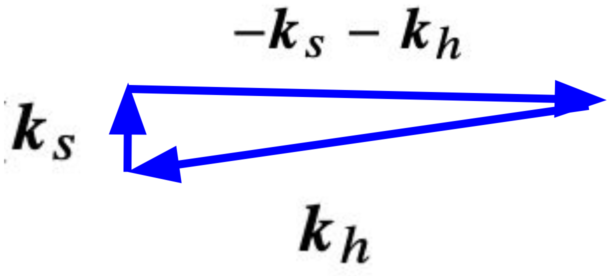
$$
\mathcal{B}_{\pm,\mathrm{fgh}}(\boldsymbol{l}) = \int \mathrm{d}\chi \frac{q_{\mathrm{f}}(\chi)q_{\mathrm{g}}(\chi)q_{\mathrm{h}}(\chi)}{\chi^4} \int \frac{\mathrm{d}^2\boldsymbol{l}_1}{(2\pi)^2} \int \frac{\mathrm{d}^2\boldsymbol{l}_2}{(2\pi)^2}
$$
$$
\times\ B_\delta^{\mathrm{3D}}\left(\frac{\boldsymbol{l}_1}{\chi}, \frac{\boldsymbol{l}_2}{\chi}, \frac{-\boldsymbol{l}_1 - \boldsymbol{l}_2}{\chi}, \chi\right) e^{2i(\phi_2 \mp \phi_{-1-2})}
$$
$$
\times\ U(\boldsymbol{l}_1)W(\boldsymbol{l}_2 + \boldsymbol{l})W(-\boldsymbol{l}_1 - \boldsymbol{l}_2 - \boldsymbol{l})\ .
$$

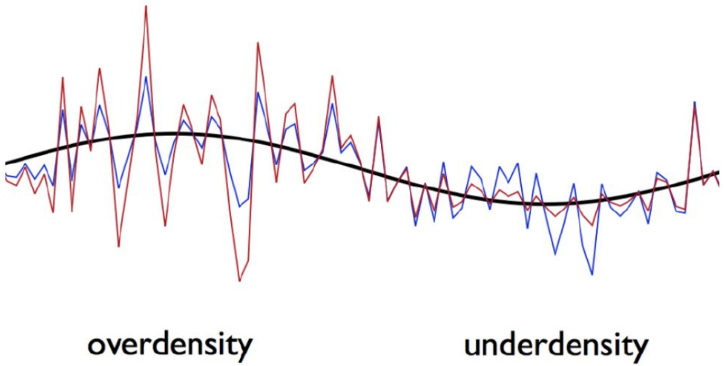
→ For *large* $\boldsymbol{l}$ values, the window functions U and W (low pass filters) impose a ***squeezed*** condition:

$$
|-\boldsymbol{l}_1 - \boldsymbol{l}_2| \approx |\boldsymbol{l}_2| \gg |\boldsymbol{l}_1|
$$



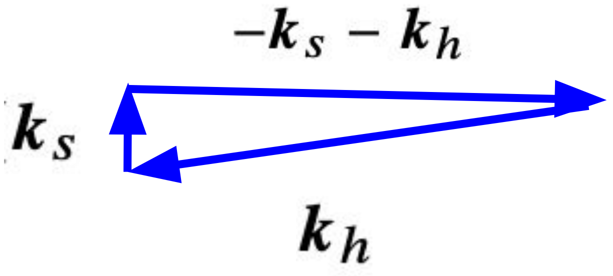
Response (RF) approach to the **squeezed** limit of the 3D matter bispectrum

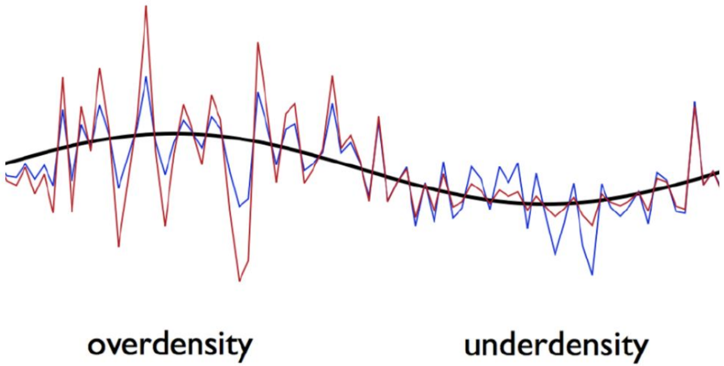




→ **Squeezed bispectrum** can be expressed in terms of **power spectrum** and its **response functions**

Response (RF) approach to the **squeezed** limit of the 3D matter bispectrum

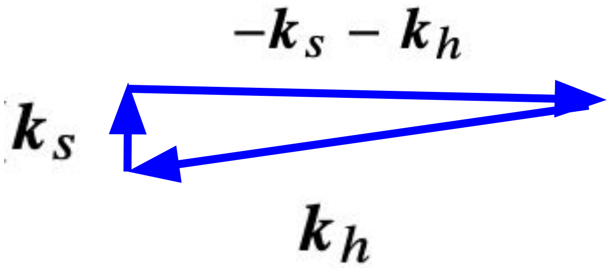


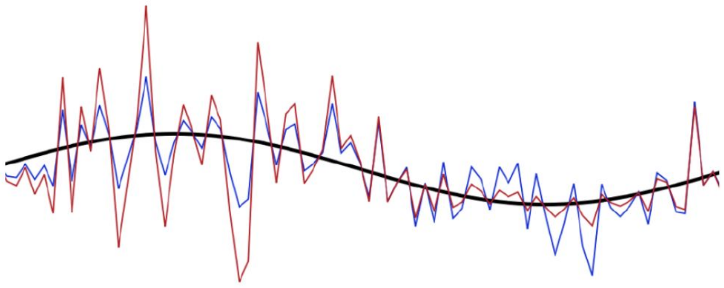


→ **Squeezed bispectrum** can be expressed in terms of **power spectrum** and its **response functions**

$$B^{\rm 3D}_{\delta,\rm RF}(\boldsymbol{k}_s, \boldsymbol{k}_h, -\boldsymbol{k}_{sh}, \tau) = \left[R_1(k_h,\tau) + \left(\mu^2_{\boldsymbol{k}_h,\boldsymbol{k}_s} - \frac{1}{3}\right) R_K(k_h,\tau)\right] P^{\rm 3D}_{\delta}(k_h,\tau) P^{\rm 3D}_{\delta,L}(k_s,\tau) + \mathcal{O}\left[\frac{k_s^2}{k_h^2}\right]$$

Response (RF) approach to the **squeezed** limit of the 3D matter bispectrum





→ **Squeezed bispectrum** can be expressed in terms of **power spectrum** and its **response functions**

$$B^{3D}_{\delta,\mathrm{RF}}(\boldsymbol{k}_s, \boldsymbol{k}_h, -\boldsymbol{k}_{sh}, \tau) = \left[R_1(k_h,\tau) + \left(\mu^2_{\boldsymbol{k}_h,\boldsymbol{k}_s} - \frac{1}{3}\right) R_K(k_h,\tau)\right] P^{3D}_{\delta}(k_h,\tau) P^{3D}_{\delta,L}(k_s,\tau) + O\left[\frac{k_s^2}{k_h^2}\right]$$

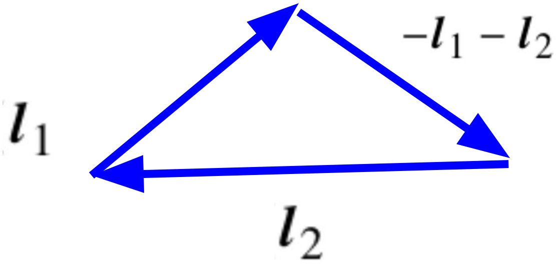
$$R_1(k,\tau) = 1 - \frac{1}{3}\frac{\mathrm{d}\ln P^{3D}_{\delta}(k,\tau)}{\mathrm{d}\ln k} + G_1(k,\tau)$$

$$R_K(k,\tau) = G_K(k,\tau) - \frac{\mathrm{d}\ln P^{3D}_{\delta}(k,\tau)}{\mathrm{d}\ln k}$$

Medium-low $\boldsymbol{l}$ values of the integrated shear bispectrum

$$\mathcal{B}_{\pm,\mathrm{fgh}}(\boldsymbol{l}) = \int \mathrm{d}\chi \frac{q_\mathrm{f}(\chi)q_\mathrm{g}(\chi)q_\mathrm{h}(\chi)}{\chi^4} \int \frac{\mathrm{d}^2\boldsymbol{l}_1}{(2\pi)^2} \int \frac{\mathrm{d}^2\boldsymbol{l}_2}{(2\pi)^2}$$
$$\times\ B_\delta^{\mathrm{3D}}\left(\frac{\boldsymbol{l}_1}{\chi}, \frac{\boldsymbol{l}_2}{\chi}, \frac{-\boldsymbol{l}_1-\boldsymbol{l}_2}{\chi}, \chi\right) e^{2i(\phi_2 \mp \phi_{-1-2})}$$
$$\times\ U(\boldsymbol{l}_1)W(\boldsymbol{l}_2+\boldsymbol{l})W(-\boldsymbol{l}_1-\boldsymbol{l}_2-\boldsymbol{l})\ .$$

→ For *medium-low* $\boldsymbol{l}$ values, the **non-squeezed** bispectrum configurations contribute



Non-squeezed configurations of the 3D matter bispectrum

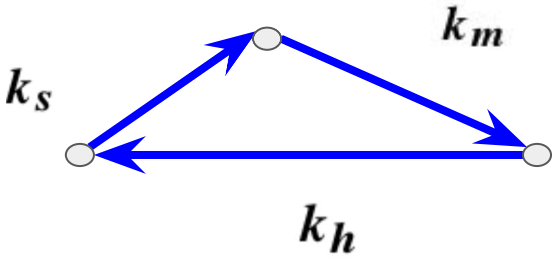
→ Use bispectrum fitting function from **Gil-Marin et al. (GM)**

$$B^{3\mathrm{D}}_{\delta,\mathrm{GM}}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3, \tau) = 2\, F_2^{\mathrm{eff}}(\boldsymbol{k}_1, \boldsymbol{k}_2, \tau)\, P_\delta^{3\mathrm{D}}(k_1, \tau)\, P_\delta^{3\mathrm{D}}(k_2, \tau) + \text{ cyclic permutations},$$

$$\begin{aligned} F_2^{\mathrm{eff}}(\boldsymbol{k}_i, \boldsymbol{k}_j, \tau) = \quad & \frac{5}{7} a(k_i, \tau) a(k_j, \tau) \\ + \; & \frac{1}{2} \mu_{\boldsymbol{k}_i, \boldsymbol{k}_j} \left(\frac{k_i}{k_j} + \frac{k_j}{k_i} \right) b(k_i, \tau) b(k_j, \tau), \\ + \; & \frac{2}{7} \mu^2_{\boldsymbol{k}_i, \boldsymbol{k}_j} c(k_i, \tau) c(k_j, \tau) \end{aligned}$$

Joint model for the matter bispectrum **(GM + RF)**

$$B_\delta^{3\mathrm{D}}(k_1, k_2, k_3, \tau) = \begin{cases} B_{\delta,\mathrm{RF}}^{3\mathrm{D}}\,, & f_{\mathrm{sq}} \geq f_{\mathrm{sq}}^{\mathrm{thr}} \implies \text{squeezed} \\ B_{\delta,\mathrm{GM}}^{3\mathrm{D}}\,, & \text{otherwise} \end{cases}$$

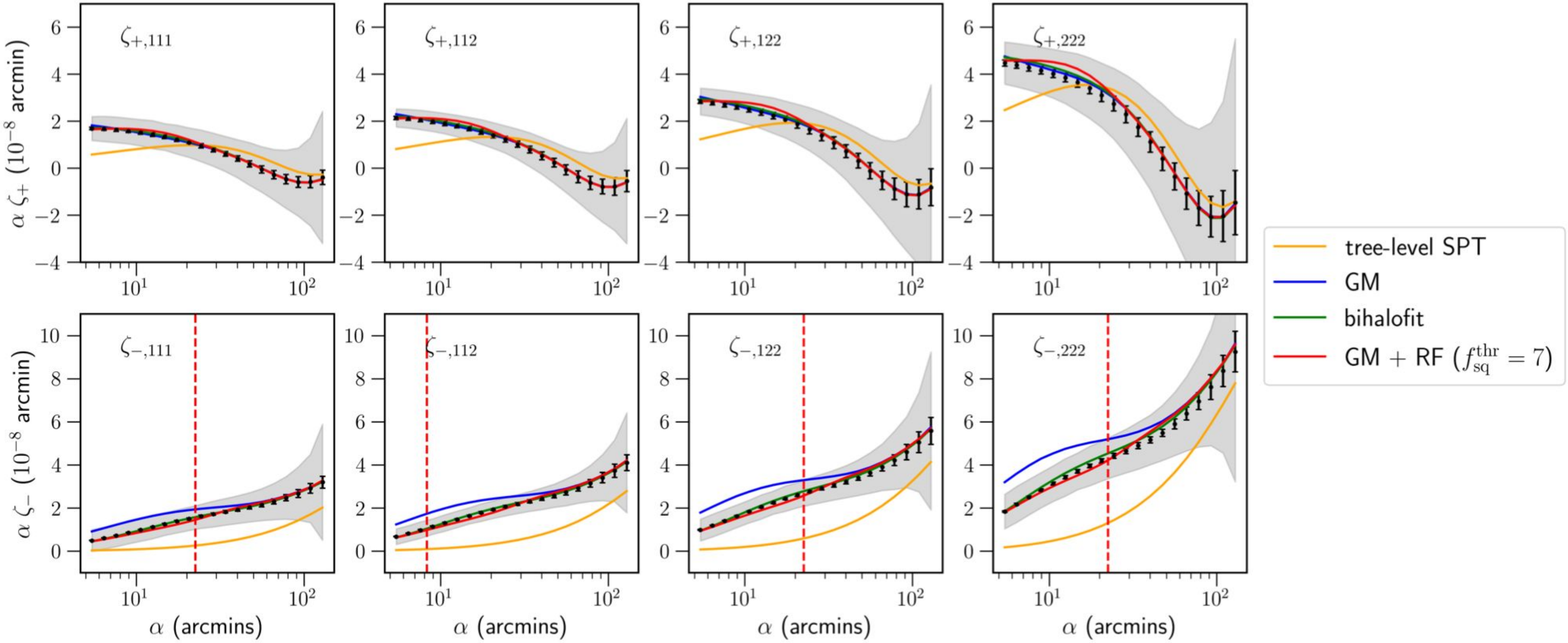


(i) Given 3 sides → arrange in descending order k_h > k_m > k_s

(ii) Define **f_sq** = k_m / k_s → **squeezeness** parameter — how squeezed a triangle is; larger the value, more squeezed it is!

(iii) If **f_sq** > some **threshold** → use RF; otherwise use GM

Comparison of different bispectra to simulation results



Encoding baryonic effects into the integrated 3PCF

The GM + RF model requires the **non-linear matter power spectrum**

- **non-squeezed branch**

$$B^{3\mathrm{D}}_{\delta,\mathrm{GM}}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3,\tau) = 2\,F_2^{\mathrm{eff}}(\boldsymbol{k}_1,\boldsymbol{k}_2,\tau)\;P^{3\mathrm{D}}_{\delta}(k_1,\tau)\;P^{3\mathrm{D}}_{\delta}(k_2,\tau)$$
$$+\;\text{cyclic permutations},$$

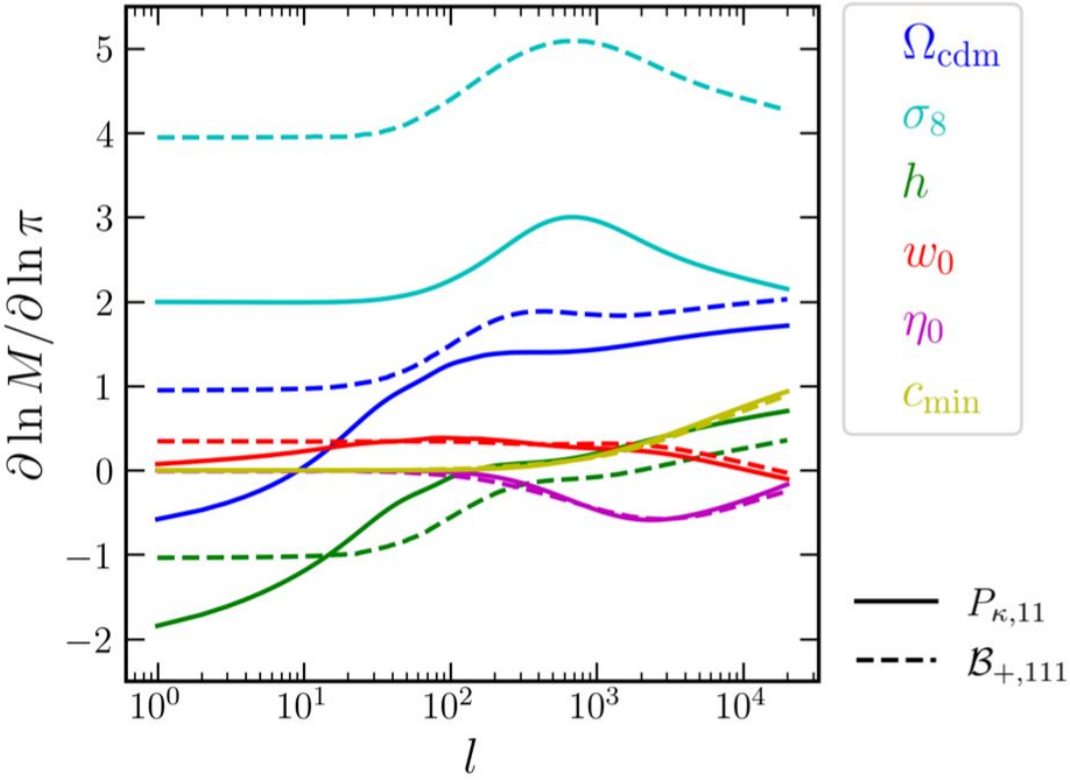
- **squeezed branch**

$$B^{3\mathrm{D}}_{\delta,\mathrm{RF}}(\boldsymbol{k}_s,\boldsymbol{k}_h,-\boldsymbol{k}_{sh},\tau) = \left[R_1(k_h,\tau) + \left(\mu^2_{\boldsymbol{k}_h,\boldsymbol{k}_s} - \frac{1}{3}\right)R_K(k_h,\tau)\right]$$
$$\times P^{3\mathrm{D}}_{\delta}(k_h,\tau)P^{3\mathrm{D}}_{\delta,L}(k_s,\tau) + O\left[\frac{k_s^2}{k_h^2}\right]$$

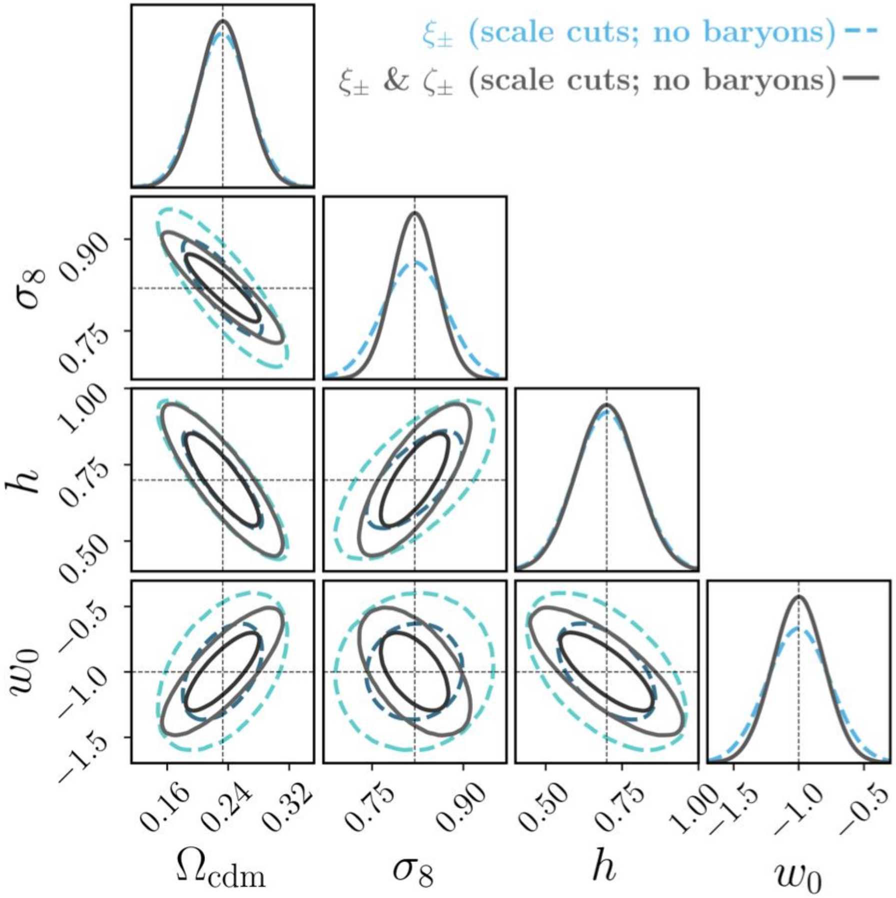
→ Encode **baryonic feedback effects** into the integrated 3PCF *through the matter power spectrum*!

→ Use HMCODE (**Mead et al. 2015**) power spectrum with **2 baryonic parameters**

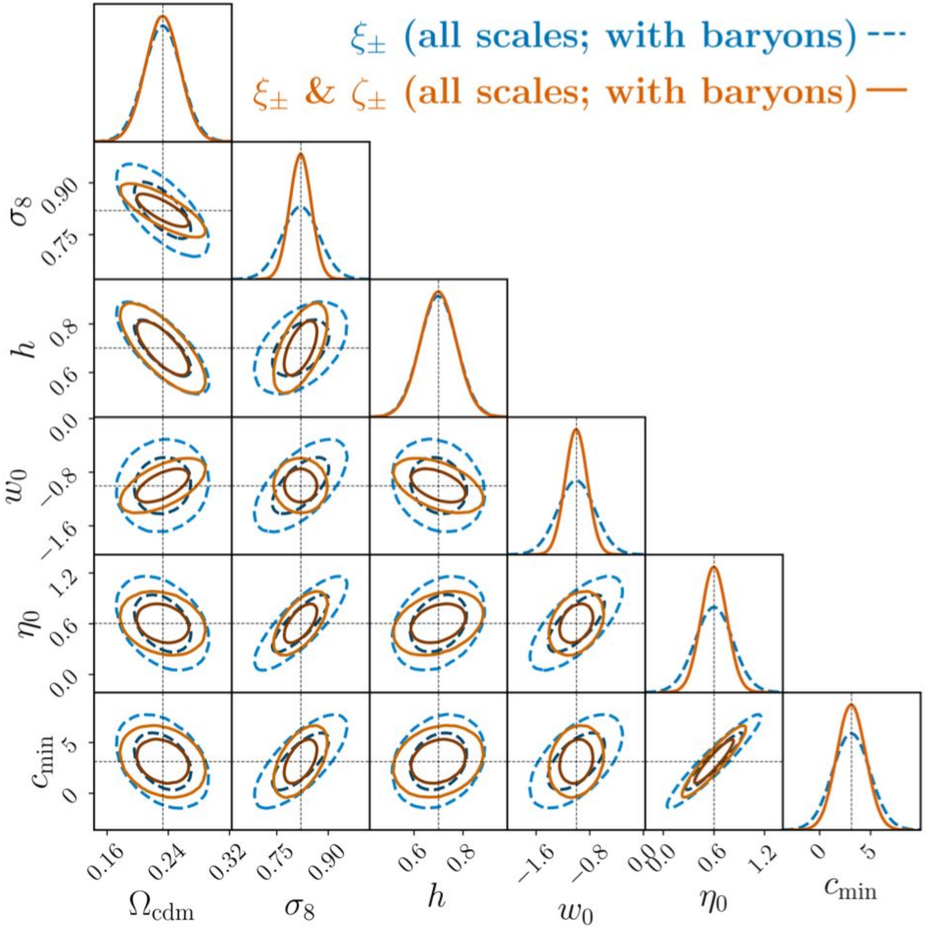
Derivatives with respect to cosmological and baryonic parameters



Fisher forecast on cosmological parameters - with scale cuts



Fisher forecast on cosmological & baryonic parameters - all scales



Conclusions and Outlook

- Integrated shear 3PCF analysis only needs the shear 2PCF and the 1-pt aperture mass measurements → easy to measure!

- Holds potential to improve constraints on cosmological as well as baryonic feedback parameters.

- **Work in progress**:
  - Optimizing the size of filters; investigating the effects of masking
  - Including systematics (photo-z, shear calibration, IA) into modelling