Can we use three-point statistics to self-calibrate weak lensing systematics?

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with Benjamin Joachimi

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UCL

I will discuss:

Motivation for considering 3-point statistics, despite their complexity

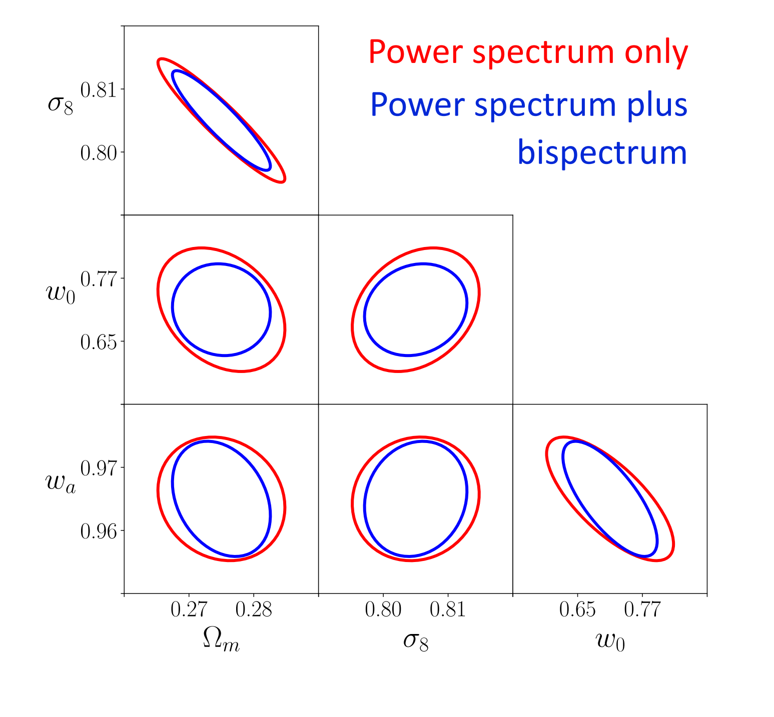
Modelling assumptions to make the problem realistic but tractable

Results, based on Fisher matrix analysis and figures of merit

Our conclusion that 3-point statistics can help a lot
+ brief discussion of further work

It is well-established that combining 2- and 3-point weak lensing statistics reduces statistical errors

Sato and Nishimichi 2013
Kayo and Takada 2013
Coulton et al 2018
Rizzato et al 2018



And 3-point statistics have been measured in practice

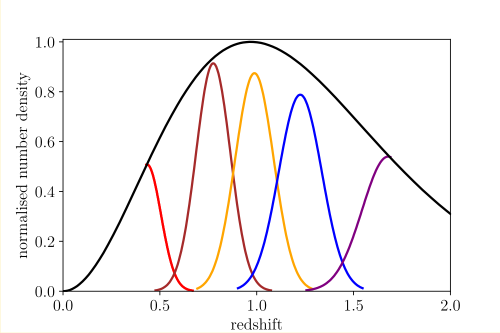
Semboloni et al 2010 – Cosmic Evolution Survey
Fu et al 2014 - CFHTLens

But the real challenge for next-generation weak lensing surveys is systematic uncertainties

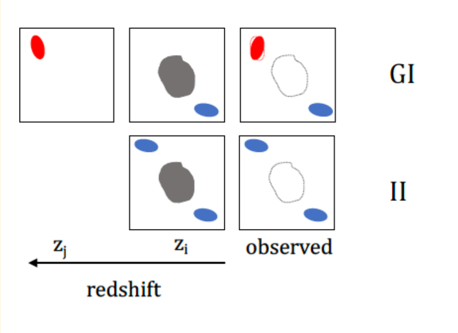
Multiplicative bias

$$\hat{\gamma}^{(i)} = (1 + m_i)\, \gamma^{(i)}_{\mathrm{true}}$$

Redshift uncertainties



Intrinsic alignments



We used Fisher matrix methods and figures of merit to quantify information content …

Fisher matrix

$$F_{\alpha\beta} = \frac{\partial \mathbf{D}^{\mathrm{T}}}{\partial p_\alpha} \mathrm{Cov}_{\mathrm{D}}^{-1} \frac{\partial \mathbf{D}}{\partial p_\beta}$$

Figure of merit

$$\mathrm{FoM}_{\alpha\beta} = \frac{1}{\sqrt{\det(\mathbf{F}^{-1})_{\alpha\beta}}}$$

… and made some simplifying modelling choices

- *Euclid*-like survey but used only 5 tomographic bins (over whole redshift range)
- Bispectrum based only on equilateral triangles
- Only Gaussian and supersample terms of covariance
  (In-survey non-Gaussian terms are sub-dominant)
- Focus on $\Omega_{\rm m} - \sigma_8$ and $w_0 - w_a$ planes

We assume multiplicative bias in each bin is independent and uncorrelated ...

$$\hat{\gamma}^{(i)} = (1 + m_i)\,\gamma^{(i)}_{\mathrm{true}}$$

Huterer et al 2006, Massey et al 2012

$$\Longrightarrow\ \hat{B}^{(ijk)}(\boldsymbol{\ell}_1, \boldsymbol{\ell}_2, \boldsymbol{\ell}_3) \approx (1 + m_i + m_j + m_k)\ B^{(ijk)}(\boldsymbol{\ell}_1, \boldsymbol{\ell}_2, \boldsymbol{\ell}_3)$$

measured BS

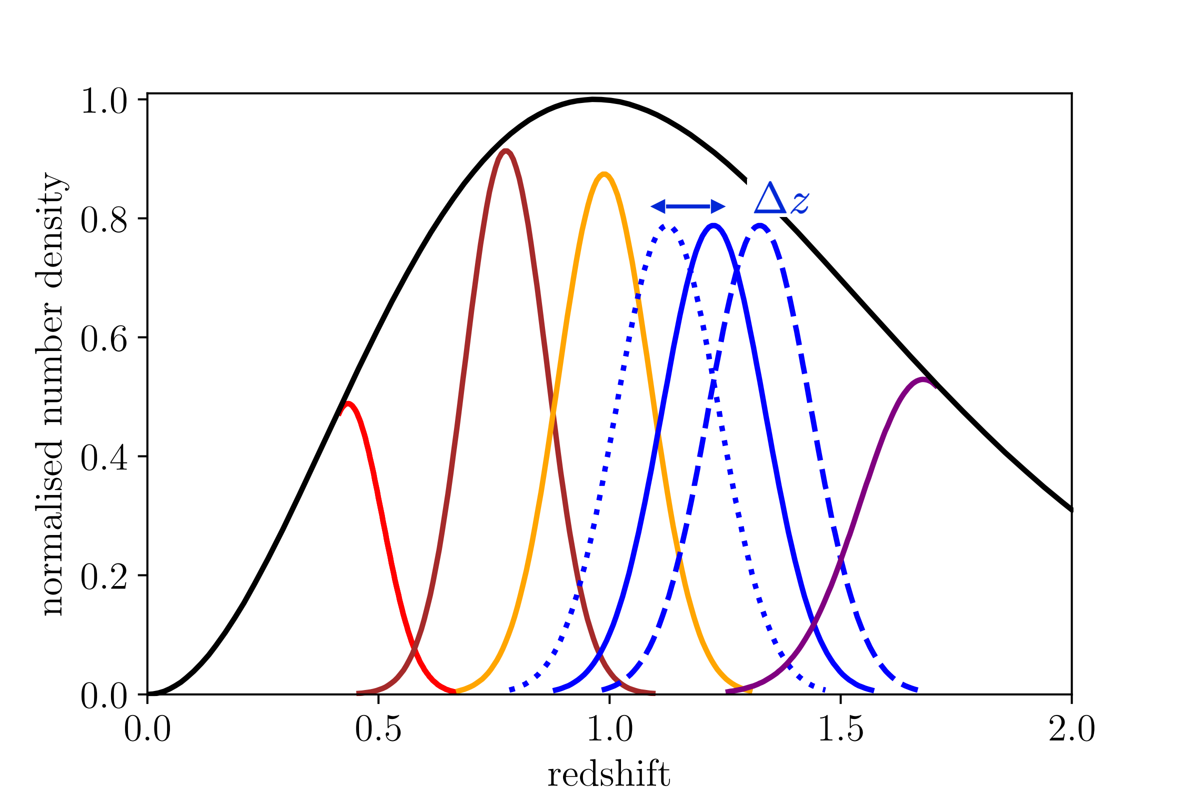
true BS

5 free parameters m_i - one for each tomographic bin

… and wrap up uncertainty in the redshift distribution into a single shift parameter for each bin

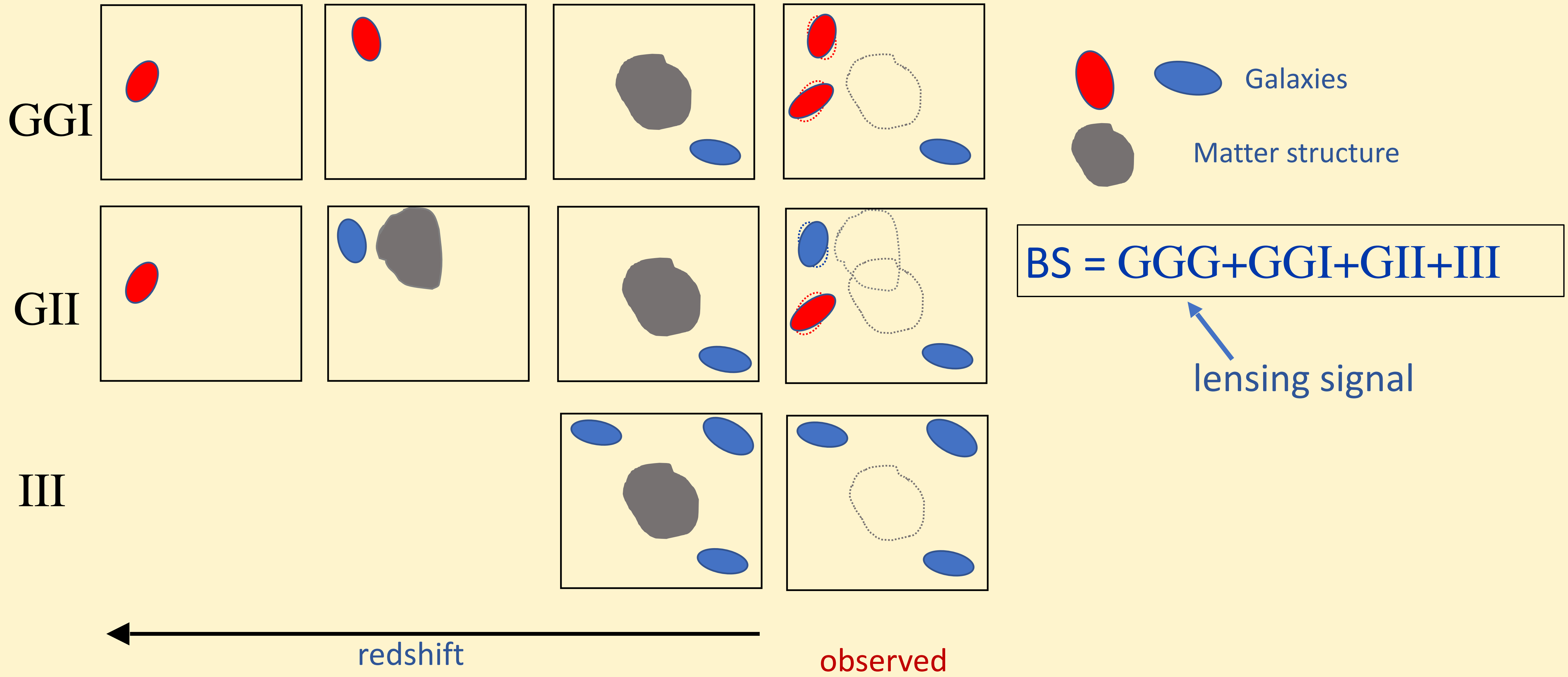
$$p^{(i)}(z) = p^{(i)}_{\rm obs}(z - \Delta z_i)$$

Again there are 5 free parameters Δz_i



Hikage et al 2019, Hildebrandt et al 2020

Intrinsic alignment bispectra are more complex than power spectra



Based on Troxel & Ishak 2015

We use the nonlinear alignment model

Fourier transform of field which produces IA → $\tilde{\delta}_{\mathrm{I}} = f_{\mathrm{IA}} \tilde{\delta}_{\mathrm{G}}$ ← matter density contrast

$$f_{\mathrm{IA}} = -A_{\mathrm{IA}} \frac{C_1 \bar{\rho}}{(1+z)D(z)} \left(\frac{1+z}{1+z_0} \right)^{\eta_{\mathrm{IA}}}$$

2 free parameters – amplitude A_{IA} and redshift dependence η_{IA}

Hirata & Seljak 2004, Bridle & King 2007

This gives the intrinsic alignment power spectra

$$P_{\delta\delta_{\rm I}}(k) = f_{\rm IA} P_{\rm NL}(k) \qquad P_{\delta_{\rm I}\delta_{\rm I}}(k) = f_{\rm IA}^2 P_{\rm NL}(k)$$

and, using the fitting formula from Gíl-Marin et al 2012:

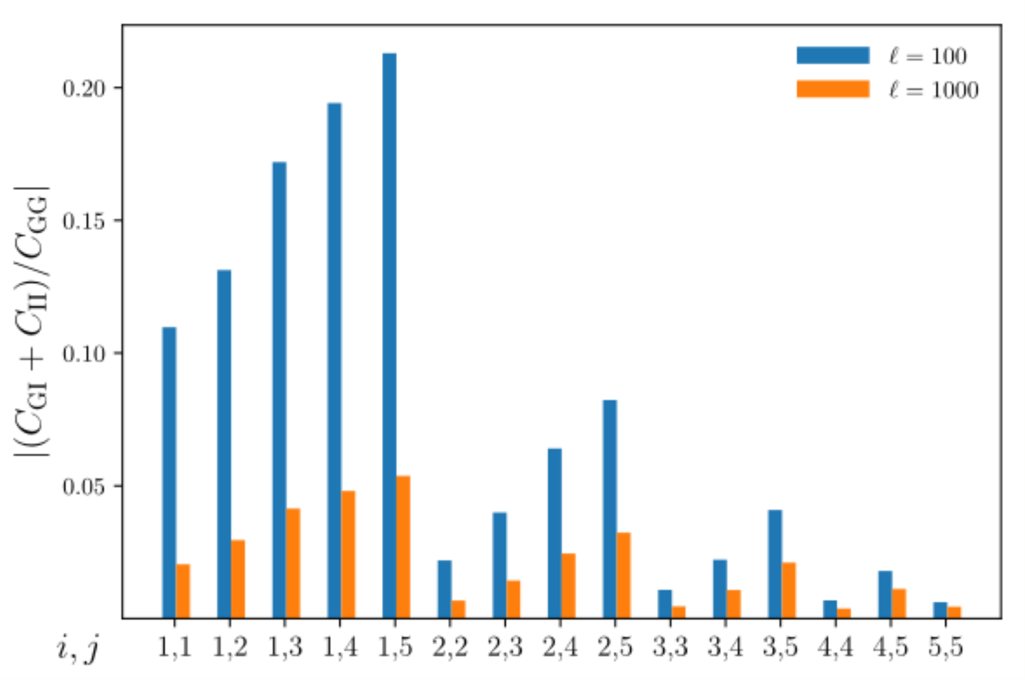
$$B_{\delta\delta\delta}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = 2F_2^{\rm eff}(\mathbf{k}_1, \mathbf{k}_2) P_{\rm NL}(k_1) P_{\rm NL}(k_2) + 2 \text{ perms.}$$

the IA bispectra, eg

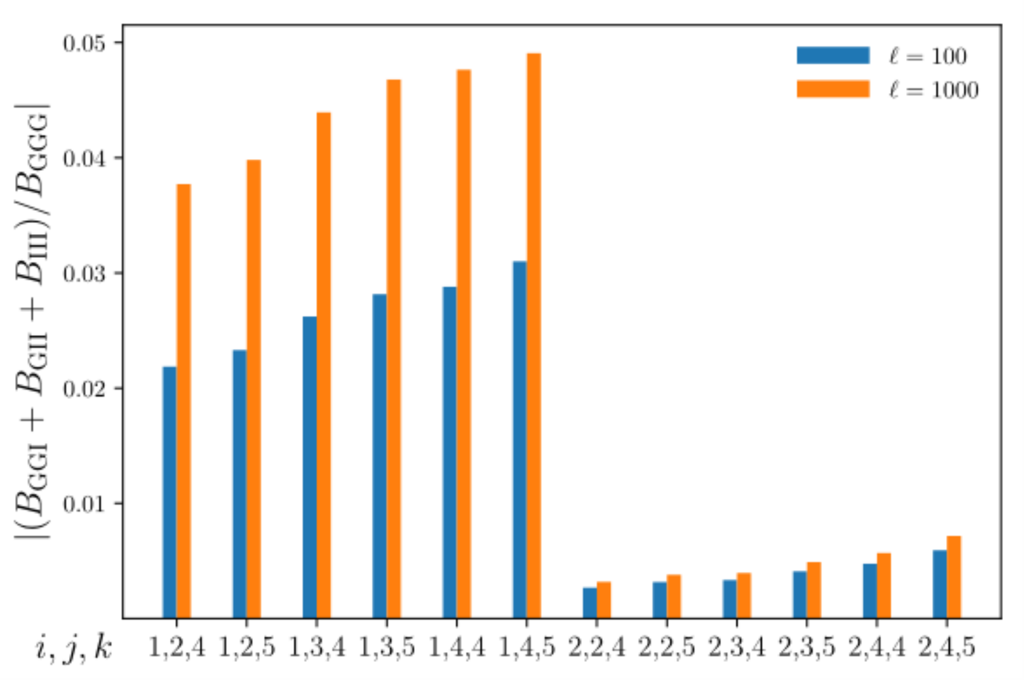
$$\begin{aligned} B_{\delta\delta_{\rm I}\delta_{\rm I}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = 2 \big[& f_{\rm IA}^3 F_2^{\rm eff}(\mathbf{k}_1, \mathbf{k}_2) P_{\rm NL}(k_1) P_{\rm NL}(k_2) \\ & + f_{\rm IA}^2 F_2^{\rm eff}(\mathbf{k}_2, \mathbf{k}_3) P_{\rm NL}(k_2) P_{\rm NL}(k_3) \\ & + f_{\rm IA}^3 F_2^{\rm eff}(\mathbf{k}_3, \mathbf{k}_1) P_{\rm NL}(k_3) P_{\rm NL}(k_1) \big] \end{aligned}$$

The resulting IA power spectra and bispectra are differently related to the lensing signal

Power spectrum



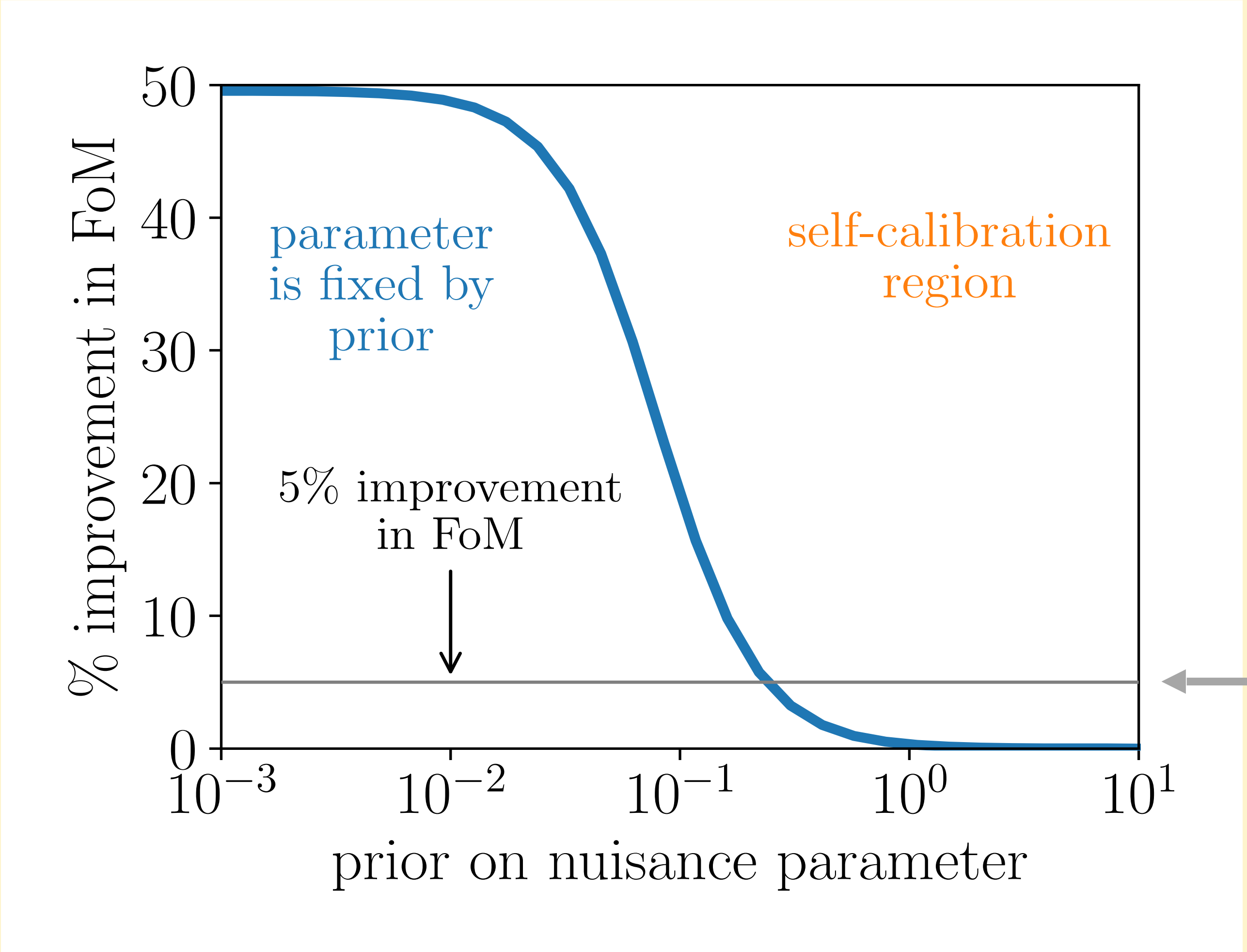
Bispectrum



Ratio of total intrinsic alignment signal to lensing signal

We consider how the FoM varies as we vary the prior on a nuisance parameter

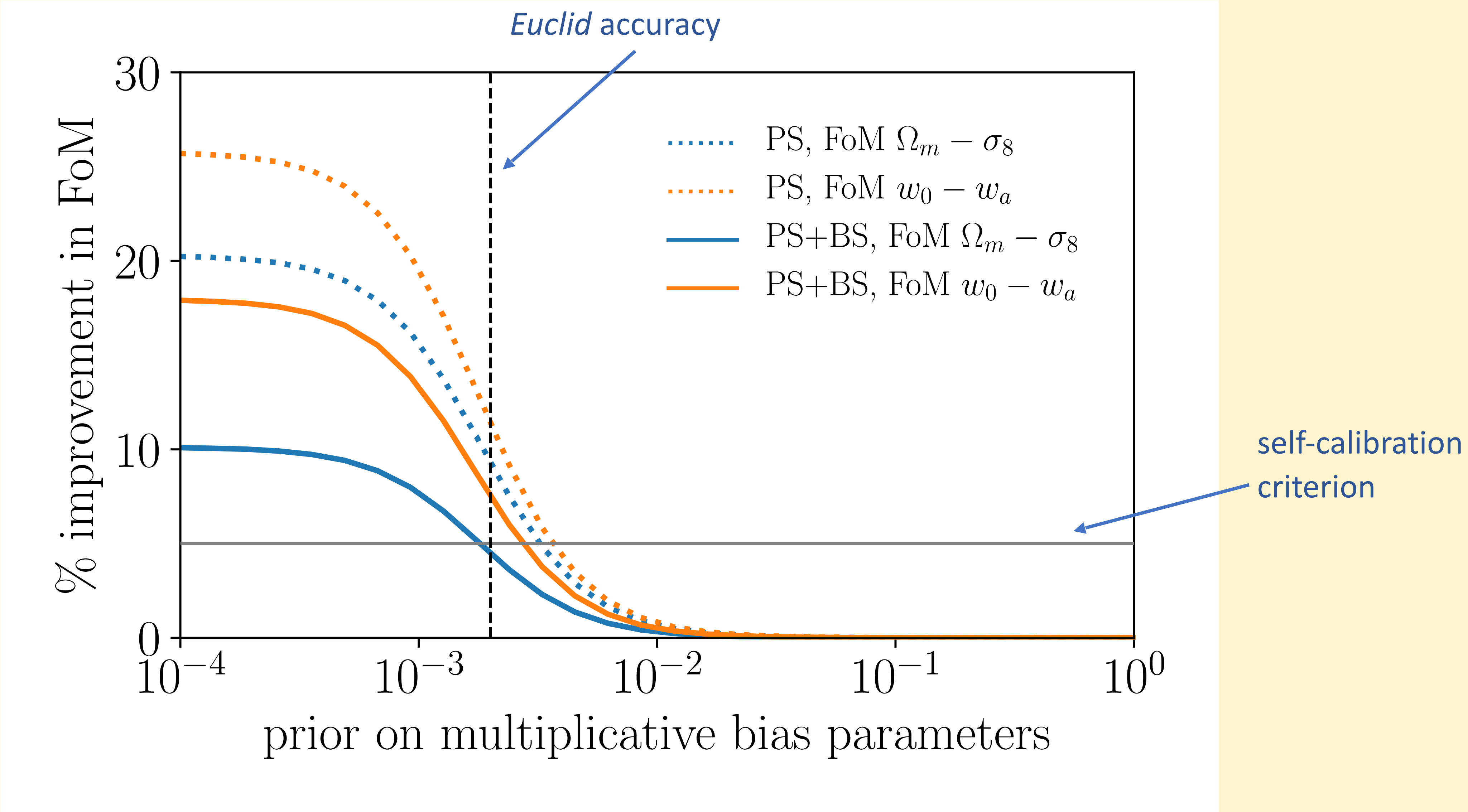
small step is good



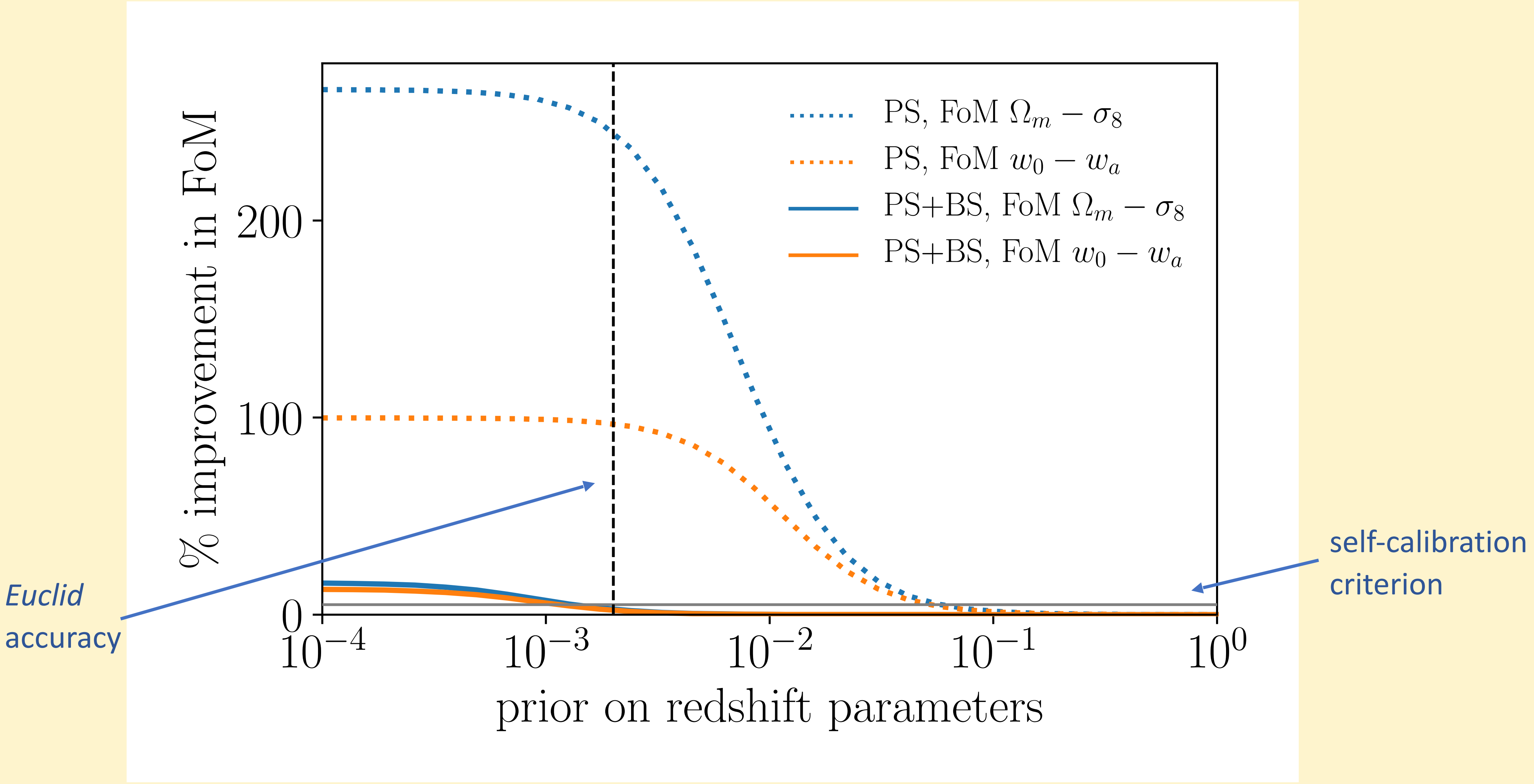
chosen definition of self-calibration

low self-calibration point is good

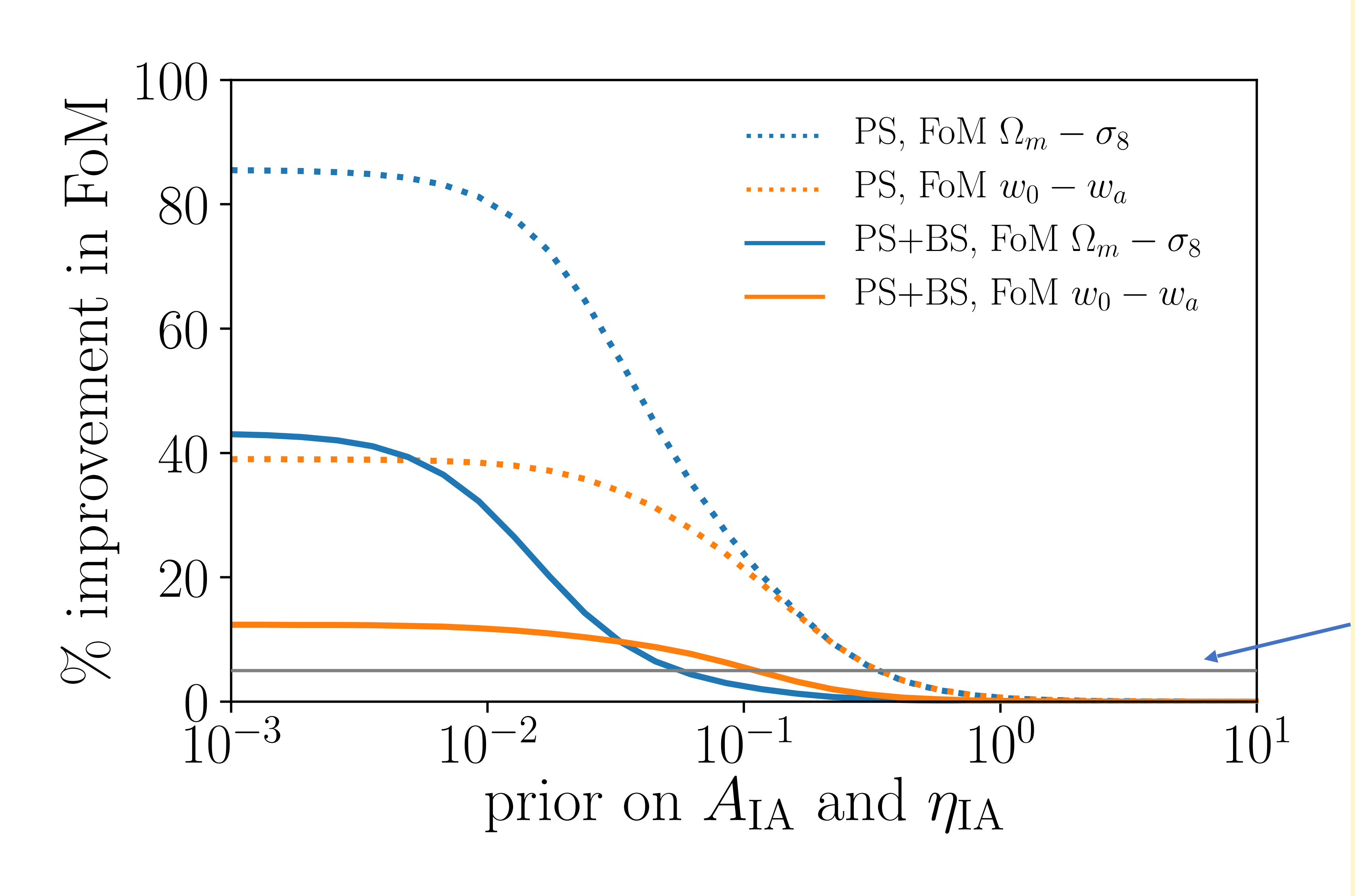
Self-calibration is possible and is improved by bispectrum – multiplicative bias



Self-calibration is possible and is improved by bispectrum – redshift uncertainty

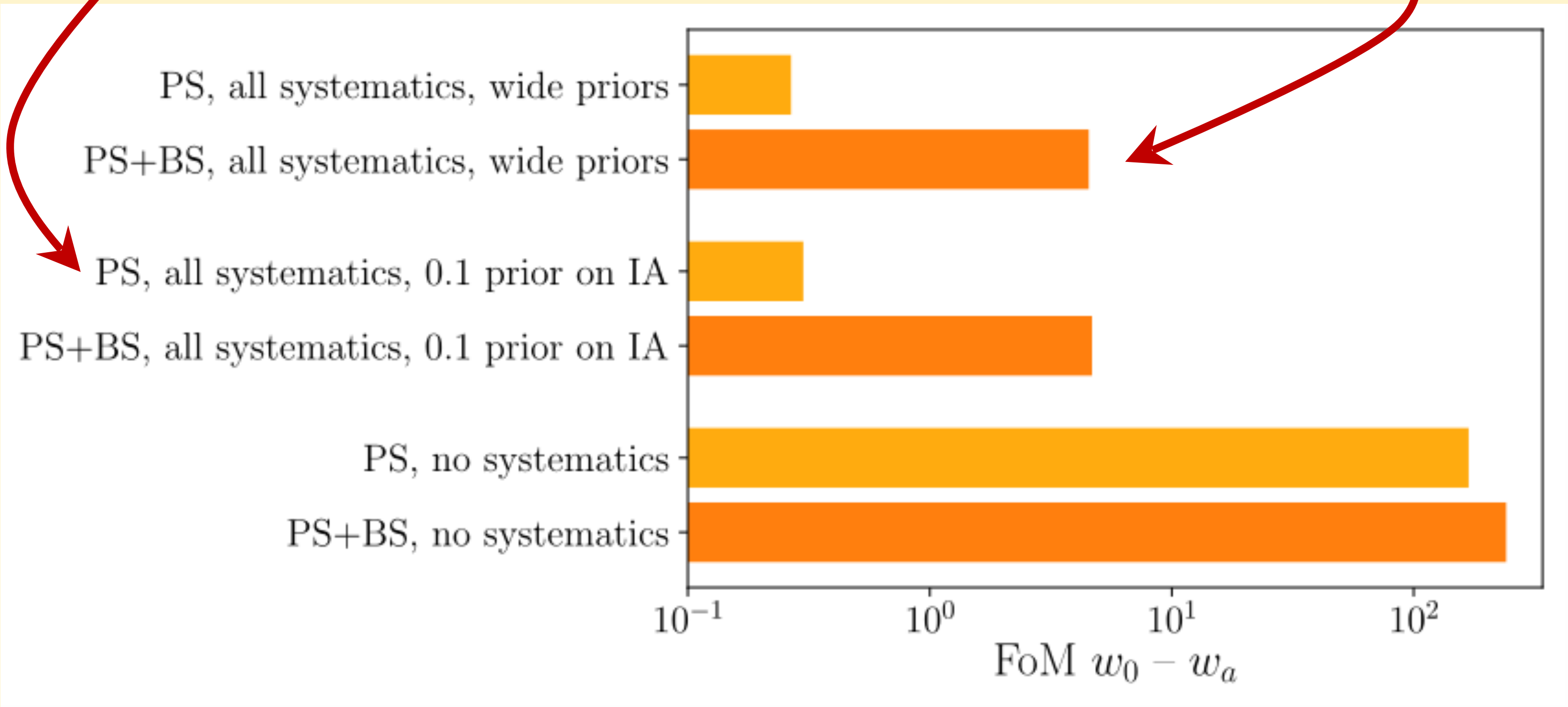


Self-calibration is possible and is improved by bispectrum – intrinsic alignments



self-calibration criterion

Another way to look at this:
compare PS with tight prior with self-calibration using PS+BS



Example: $w_0 - w_a$ FoM

Further work

- Consider alternative summary statistics which are more useful in practice eg aperture mass
- Confirm the bispectrum intrinsic alignment model – measure from simulations
- Investigate improved bispectrum formula Takahashi et al 2020 - better at small scales
- Look into other systematics, especially baryonic effects - compare with established power spectrum methods (Mead et al 2021)

Summary

- Systematics are a key challenge for next-generation weak lensing surveys
- Systematics affect the power spectrum and bispectrum differently
- Have shown that using the bispectrum allows self-calibration to mitigate three major systematics
- Hopefully this will lead to a practical alternative method for future surveys